SPACE-TIME AS STRONGLY BENT PLATE

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Abstract

Futher development is made of a consept of space-time as multidimensional elastic plate, proposed earlier in [20, 21]. General equilibrium equations, including 4-dimensional tangent stress tensor — energy-momentum tensor of matter — are derived. Comparative analysis of multidimensional elasticity theory (MET) and GR is given. Variational principle, boundary conditions, energy-momentum tensor, matter and space-time signature are reviewed within the context of MET.

1 Introduction

It seems likely Cliffords works [1] laid philosophic foundations of modern space-time physics. His ideas about physical properties of space have been realized in GR and in a number of more recent works on its alternative formulations and generalizations. Part of this works has used analogy between GR and *elasticity theory*, noted by Born [2]. The formal analogy between metric and strain tensor and the idea of elasticity of space-time have been discussed in [3]-[7].

The present paper joines the idea of elasticity with a consept of *multidimensional space-time.* This last is used in majority of modern unified theories (Kaluza-Klein models [8, 9, 10], strings and p-branes physics [11], supersymmetry [12], quantum field theory in bundle space [13] and others) in one way or another. The matter of present paper corelates with multidimensional theories, appealing to an idea of *embedding space.* Embedding theory [14] has been employed by many authors in attempts of physical meaning of GR clearing [15]-[19].

In works [20, 22] it has been proposed to consider space-time as multidimensional elastic body, whose sizes along four dimensions much greater, then along another ones, so that this last are macroscopically unobservable. Such body has been called *4-dimensional*

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plate. State of the plate can be described by multidimensional free elastic energy, depending on mechanical strain of the plate. Inner geometry, induced by the strain appears as geometry on the plates surface, which can be easily obtained by standard methods of embedding theory [14]. In work [20] the case of a weak bending has been investigated. Equilibrium equations, derived by variational method have been disolved in [22] for the case of plane symmetry.

Preliminary comparing of the approach with standard GR implies the hypothesis, which is supported by following consideration: *gravity, generating curvature of space-time, is manifestation of normal bending of the plate. Its energy is concentrated in microscopic thicknesses of the plate. Matter and fields appear as tangent to the plate surface stresses and their energy is macroscopic energy of stretches and shears of the 4-dimensional cover.* In other words *there is no space-time and matter separately, but we have unified space-time-matter object*[1]. The approach reveals ideas, proposed by many other authors [1]-[7] and [15]-[19].

The plan of this paper is as follows. First part is devoted to a generalization of previous results, obtained in [20].

General mathematical and physical setting of the problem is given in Sec.2.

In Sec.3 previous results of [20] are explicitly compared with linearized Einstein theory.

In Sec.4 general equilibrium equations are derived and in Sec.5 meaning of strain weakness conditions is cleared out.

Second part of the paper containes review of well known objects of standard field theory in terms of multidimensional elasticity theory (MET).

In Sec.6 mechanical interpreting of field Lagrangian is proposed. Example of classical newtral scalar field is considered.

In Sec.7 conservation laws and its origin are considered from the view point of MET.

Sec.8 is devoted to a question of true variational variables.

In Sec.9 notions of energy-momentum tensor, matter and its density are analysed within the context of MET. New understanding of signature of space-time is given.

Sec.10 is devoted to the problems of boundary terms and boundary conditions.

In Conclusion we make short discussion.

2 Setting of the problem

Lets consider pseudoeuclidian vector space $M_{p+1,q+3}$ of $N+4$ dimensions[2], $p+1$ of which are time-like and $q+3$ — space-like, $p+q=N$. Span of four arbitrary linear independent

[1] The similar idea has been proposed within Kaluza-Klein theory in [9, 10] for different reasons and by quite different way.

[2] Here and below we let big English letters represent $\overline{1, N+4}$, small — $\overline{1, N}$ and Greek — $0, 1, 2, 3$, unless otherwise specified.

vectors $\{t_{(\mu)}\}$, satisfying the conditions[3]:

$$\Theta(t_{(0)}, t_{(0)}) > 0,\ \Theta(t_{(i)}, t_{(i)}) < 0, \quad (i = 1, 2, 3), \tag{1}$$

where Θ — metric in $M_{p+1,q+3}$, forms 4-dimensional Minkowski plane $M_{1,3}$ and gives global decomposition[4] of $M_{p+1,q+3}$: $M_{p+1,q+3} = M_{1,3} \oplus M_{p,q}$, where $M_{p,q}$ — orthogonal ajunct of $M_{1,3}$ to original $M_{p+1,q+3}$. Corresponding decomposition for metric has the form: $\Theta = \theta + \bar{\theta}$, where θ — metric in $M_{1,3}$ and $\bar{\theta}$ — metric in $M_{p,q}$, so that $\bar{\theta}(u,) = 0$ and $\theta(v,) = 0$ for any vectors $u \in M_{1,3},\ v \in M_{p,q}$.

Let $\{X^A\}$ be the set of affine coordinates[5] in $M_{p+1,q+3}$, concordant with the decomposition of $M_{p+1,q+3}$: $X = x \oplus \bar{x}$, where $\{x^\mu\}$ and $\{\bar{x}^m\}$ — the sets of coordinates in $M_{1,3}$ and $M_{p,q}$ correspondingly, satisfying the condition

$$\Theta(dx, d\bar{x}) = 0; \quad \theta(d\bar{x}, d\bar{x}) = 0; \quad \bar{\theta}(dx, dx) = 0, \tag{2}$$

Then equation of $M_{1,3}$ will take the simple form:

$$\bar{x} = 0. \tag{3}$$

and line element in $M_{p+1,q+3}$ due to conditions (2), takes the form:

$$ds^2 = \Theta(dX, dX) = \Theta(dx + d\bar{x}, dx + d\bar{x}) = \theta(dx, dx) + \bar{\theta}(d\bar{x}, d\bar{x}). \tag{4}$$

Without loss of generality metrics θ and $\bar{\theta}$ can be put to diagonal form with ± 1 on diagonals:

$$\|\theta\| = \mathrm{diag}(1, -1, -1, -1); \quad \|\bar{\theta}\| = \mathrm{diag}(\varepsilon_1, \ldots, \varepsilon_N); \quad \varepsilon_i = \pm 1$$

by independent linear transformations of $\{x^\mu\}$ and $\{\bar{x}^m\}$ and these last may be regarded as cartesian.

We postulate, that *geometrical Minkowski plane $M_{1,3}$ is middle plane of 4-dimensional physical elastic plate*, which models classical macroscopic Minkowski space-time of special relativity. So, we endow $M_{1,3}$ by the set of thicknesses $\{h_m\}$ in extradimensions, lied in $M_{p,q}$, and by multidimensional phenomenological elastic constants — Lame coefficients (see (8)).

Let consider smooth[6] vector field Ξ, defined on $M_{1,3}$: $\Xi = \Xi(x)$. This field, if it is nonlinear function of x, gives transformation of $M_{1,3}$: $M_{1,3} \xrightarrow{\Xi} V_{1,3}$, where $V_{1,3}$ — riemannian space, which *models macroscopic space-time of GR*. Such transformation is deformation,

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[3]In present article for compactness and generality sakes we use coordinateless representation of tensors and tensor equations. All tensor should be understood as polylinear functionals, acting in corresponding vector and form spaces. Because of metrics presence we don't difer vectors and forms by special notations, unless otherwise specified.

[4]We postulate, that topology of $M_{p+1,q+3}$ is trivial.

[5] The $\{X^A\}$ forms vector X with respect to affine group of transformations in $M_{p+1,q+3}$.

[6]Requirement $\Xi \in C^4$ is sufficient.

which from physical point of view can be associated with mechanical *straining* of the plate[7]. So Ξ can be treated as displacement vector field of standard elasticity theory [23], $V_{1,3}$ — as middle plane of strained plate and $M_{1,3}$ — as coordinate map of $V_{1,3}$. Position of any point of $V_{1,3}$ in $M_{p+1,q+3}$ after straining is described by the formula:

$$X' = X + \Xi$$

or, using (3), in splitting form

$$x' = x + \xi; \quad \bar{x}' = \bar{\xi},$$

where ξ and $\bar{\xi}$ — are tangent and normal to $M_{1,3}$ components of Ξ.

Line element (4) on the plate surface with using (2) can be transformed to

$$ds'^2 = \Theta(dX + d\Xi, dX + d\Xi) = ds^2 + 2\mathcal{D}(dx, dx) = g(dx, dx), \tag{5}$$

where

$$\mathcal{D} = \partial\dot{\otimes}\xi + \frac{1}{2}(\partial' \otimes \partial'')(\theta(\xi', \xi'') + \bar{\theta}(\bar{\xi}', \bar{\xi}'')) \tag{6}$$

— is symmetric *strain tensor*, $g = \theta + 2\mathcal{D}$ — metric on $V_{1,3}$. We have used following notations: ∂ — partial derivative operator in $M_{1,3}$, $a\dot{\otimes}b \equiv (1/2)(a\otimes b + b\otimes a)$ — symmetric tensor product, $'$ and $''$ denote arguments of partial derivatives. In coordinate form (6) is:

$$\mathcal{D}_{\mu\nu} = \xi_{(\mu,\nu)} + \frac{1}{2}\xi^{\lambda}_{,\mu}\xi_{\lambda,\nu} + \frac{1}{2}\bar{\xi}^{m}_{,\mu}\bar{\xi}_{m,\nu}, \tag{7}$$

where Greek indices are contracted with the help of metric θ, small latin indices — with the help of $\bar{\theta}$.

We have formulated kynematic part of the problem. At the first step of dynamic part we postulate multidimensional *Hookes law*. In other words, we take free elastic energy density in the form:

$$\mathcal{F} = \mu\mathcal{D}\cdot\mathcal{D} + \frac{\lambda}{2}\theta^2(\mathcal{D}) \tag{8}$$

where μ, λ — phenomenological multidimensional *Lame coefficients*, $\mathcal{D}\cdot\mathcal{D} \equiv \mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\nu}$ — tensor "scalar product", $\theta(\mathcal{D}) \equiv \mathcal{D}^{\mu}_{\mu}$ — trace of $\mathcal{D}$. In standard elasticity theory the constants E — *Youngs modulus* and σ_P — *Poisson coefficient* are used more often, because of their physical clearness. They are related with Lame coefficients by the formulas (see [20], also [7]):

$$\mu = \frac{E}{2(1+\sigma_P)}; \quad \lambda = \frac{E\sigma_P}{(1+\sigma_P)(1-(n-1)\sigma_P)}; \tag{9}$$

In our case[8] $n = N + 4$.

[7] Here we don't pose the question about nature of multidimensional straining forces.

[8] Under $n = 3$ we get standard formulas of [23].

Direct generalization of standard thermodynamics gives

$$d\mathcal{F} = -\mathcal{S}dT + \sigma \cdot d\mathcal{D} \tag{10}$$

where $\mathcal{S}$ and T — multidimensional entropy density and temperature of plate substance, σ — *stress tensor*. Its alternative definition can be expressed by the integral

$$\oint\limits_{\partial\Sigma} \sigma(\ ,d\,\mathrm{vol}[\partial\Sigma]) = f_\Sigma \tag{11}$$

where f_Σ — multidimensional force vector, acting on finite arbitrary volume Σ inside elastic body, $d\,\mathrm{vol}[\partial\Sigma]$ — suitable element of boundary hyphersurface $\partial\Sigma$, oriented out of Σ. Expressions (10) and (11) give equivalent σ if last is symmetric, that can be accepted without loss of generality (see [23]). Divergence of σ determines local volume force density f:

$$\mathrm{div}\,\sigma = f, \tag{12}$$

where $\mathrm{div}\,\sigma_\alpha = \partial^\beta\sigma_{\alpha\beta}$. As it is in standard theory expression (10) gives the rule for calculation of stress tensor

$$\sigma = \left(\frac{\partial\mathcal{F}}{\partial\mathcal{D}}\right)_T. \tag{13}$$

Taking differential of (8)

$$d\mathcal{F} = 2\mu\mathcal{D} \cdot d\mathcal{D} + \lambda\theta(\mathcal{D})d\theta(\mathcal{D})$$

and substituting it into (13) we obtaine

$$\sigma = 2\mu\mathcal{D} + \lambda\theta(\mathcal{D})\theta \tag{14}$$

— *multidimensional Hookes law.*

If Π is vector force density, acting at the bound $\partial\Sigma$ of multidimensional elastic body, then boundary conditions for σ are:

$$\sigma(\ ,\tau)|_{\partial\Sigma} = \bar{P}; \quad \sigma(\ ,n)|_{\partial\Sigma} = P, \tag{15}$$

where $n, \tau, P, \bar{P}$ — normal and tangent to $\partial\Sigma$ unit vectors and components of Π correspondingly. As in standard theory we assume, that external bending stresses are much less then internal compensating ones. In other words we put:

$$\sigma(\ ,\tau)|_{\partial\Sigma} = \sigma(\ ,n)|_{\partial\Sigma} = 0. \tag{16}$$

Note, that all differentiations and integrations can be expressed through internal coordinates x' of the plate surface by changing $\partial \to \nabla$ and by definition of suitable volume form, for example: $d\,\mathrm{vol}[\Sigma] = \sqrt{\det[g]}dx'^0 \wedge dx'^1 \wedge dx'^2 \wedge dx'^3$. Fortunately this is not necessary, since all corrections are of higher smallness and can be ommited in linear elasticity theory.

All relations (6)-(16), being general in its form, have, as a matter of fact, 4-D sense. Particulary, (8) expressed through (7) gives only 4-dimensional part of free energy, induced by 4-dimensional stretches and shears of the plate, which yet can be regarded as elastic 4-dimensional cover without thicknesses. By these reasons we shall use notation $\mathcal{F}_s$ instead of $\mathcal{F}$ to difer it from $\mathcal{F}_b$ — energy density of *pure bending*.

3 Previous results

To take into account free energy of pure bending it is necessary to investigate stress distribution along extradimensions. It has been carried out in [20]. We present final results without intermediate calculations.

Free energy of bending, calculated with using (16) and integrated over thicknesses of the plate has the form

$$F_b = \frac{\mu H_N}{12} \int\limits_{\Sigma} \left\{ \bar{\theta}(\partial^2 \bar{\xi}_h \cdot \partial^2 \bar{\xi}_h) + f \bar{\theta}(\theta(\partial^2 \bar{\xi}_h), \theta(\partial^2 \bar{\xi}_h)) \right\} d\,\mathrm{vol}[\Sigma], \tag{17}$$

where $H_N = \prod\limits_{i=1}^{N} h_i$, factor $f = \lambda/(N\lambda + 2\mu)$, ∂^2 — tensor second derivative operator: $[\partial^2 f(x)]_{\alpha\beta} \equiv \partial_\alpha \partial_\beta f(x)$, $\bar{\xi}_h = \{\bar{\xi}^m\} \times \{h_m\} \equiv \{\bar{\xi}^m h_m\}$ (without summation), Σ — 4-dimensional region of the plate. Note, that (17) does'nt contain ξ-component, which can be ommited under weak bending (see [23]).

Variation of (17) over $\bar{\xi}$ (under condition $\delta(d\,\mathrm{vol}[\Sigma]) \approx 0$) is

$$\delta F_b = \int\limits_{\Sigma} \bar{\theta}(\Box^2 \bar{\xi}_D - \bar{P}, \delta\bar{\xi})\, d\,\mathrm{vol}[\Sigma] - \oint\limits_{\partial\Sigma} \bar{\theta}(\theta(\Box \partial \bar{\xi}_D, d\,\mathrm{vol}[\partial\Sigma]), \delta\bar{\xi}) \tag{18}$$

$$+ \frac{1}{f+1} \oint\limits_{\partial\Sigma} [f \bar{\theta}(\Box \bar{\xi}_D, \theta(d\,\mathrm{vol}[\partial\Sigma], \delta\partial\bar{\xi})) + \bar{\theta}(\partial^2 \bar{\xi}_D (d\,\mathrm{vol}[\partial\Sigma], \delta\partial), \bar{\xi})]$$

where $\Box = \theta(\partial, \partial)$ — wave operator, $\bar{\xi}_D = \{\bar{\xi}^m\} \times \{D_m\} = \{\bar{\xi}^m D_m\}$ (without summation),

$$D_m = \frac{\mu H_N h_m^2 (f+1)}{6} = \frac{E H_N h_m^2}{12(1+\sigma_P)} \frac{1 + \sigma_P(N-n+2)}{1+\sigma_P(N-n+1)} \tag{19}$$

— constant cylindrical stiffness factor of the plate in m-th extradimension. In our case $n = N + 4$[9]. $\bar{P}$ — external bending force density, satisfying $\theta(\bar{P},\ ) = 0$ and

$$\delta U = - \oint\limits_{\Sigma} \bar{\theta}(\bar{P}, \delta\bar{\xi}) d\,\mathrm{vol}[\Sigma],$$

where U — multidimensional potential energy. Last two integrals in (18) are boundary terms, which must be taken into account to obtaine unique solution. Equilibrium

[9] For $n = 3$ $N = 1$ we have the standard formulae [23]

equations, derived from extremum condition $\delta(F_b+U)=0$ are inhomogeneous bewave equations

$$\Box^2\bar{\xi}_D=\bar{P} \tag{20}$$

— direct generalization of Sophy-Germain beharmonic equation: $D\Delta^2\xi=P$ of standard elasticity theory [23].

Using dimensional arguments, one can get the following remarkable relation between multidimensional parameters and Einstein gravitational constant æ:

$$Eh^{N+3}\sim 1/æ.$$

This relation is in line with Sacharov's hypothesis of possible elasticity of space-time [3]. Another variants of possible relations see in [20].

In case $f=-1$ ($\sigma_P=1/2$) integrand bracket in (17) become similar to $\mathcal{R}_s$ — scalar curvature of $V_{1,3}$, expressed through second derivatives $\partial^2\bar{\xi}$ with using embedding theory [14]. In [20] this fact has been interpreted as correspondence between the proposed approach and linearized Einstein theory. It can be checked by the straightfoward way.

Away from a bodies, gravitational field can be described by linearized theory of gravitation, wherein metric tensor is the sum of a flat Minkowski metric and its small disturbance ψ [24]:

$$g=\theta+\psi. \tag{21}$$

In this case one may reject qudratic over Cristoffel symbols part of expression for curvature tensor ${}^4\mathcal{R}$ and hold only linear over second derivatives $\partial^2 g$ one, that leads to the expression:

$${}^4\mathcal{R}^{\mathrm{lin}}(a,b,c,d)=\frac{1}{2}(\partial^2\psi(a,d,b,c)+\partial^2\psi(b,c,a,d)-\partial^2\psi(a,c,b,d)-\partial^2\psi(b,d,a,c)), \tag{22}$$

where $\partial^2\psi(a,b,c,d)\equiv\partial_\alpha\partial_\beta\psi_{\gamma\delta}a^\gamma b^\delta c^\alpha d^\beta$ and a,b,c,d — arbitrary vectors. Then, using expression (5) for riemannian metric of strained plate we get ψ in the form:

$$\psi=\frac{1}{2}\bar{\theta}(\partial\bar{\xi},\partial\bar{\xi}) \tag{23}$$

where we have ommited addends with ξ in (5).

Substituting this expression into (22) and making elementary transformations we have

$${}^4\mathcal{R}^{\mathrm{lin}}(a,b,c,d)=(\bar{\theta}(\partial^2\bar{\xi}(a,c),\partial^2\bar{\xi}(b,d))-\bar{\theta}(\partial^2\bar{\xi}(a,d),\partial^2\bar{\xi}(b,c)), \tag{24}$$

that after appropriate contractions with flat metric θ (and rescaling $\bar{\xi}\to\bar{\xi}_h$) gives (up ty a sign) the integrand bracket of (17) under $f=-1$.

So, multidimensional Hookes law, applied for describing of weakly bent 4-dimensional plate under special value of elastic constants corresponds to linearized GR.

4 General equilibrium equations

To consider non weak gravitational and another 4-dimensional fields one should take into account both free energy F_b of pure bending and free energy F_s of 4-dimensional stretches-shears. Our derivation of general equilibrium equation follows to [23] with suitable generalizations and addends.

Total elastic free energy F is the sum: $F = F_s + F_b$, where second term is (17). Stretch energy F_s can be obtained from (8) by integrating over plate volume. Using change $d\,\mathrm{vol}[M_{p+1,q+3}] \to H_N d\,\mathrm{vol}[\Sigma]$ we get

$$F_s = H_N \int\limits_\Sigma \mathcal{F}_s\, d\,\mathrm{vol}[\Sigma]. \tag{25}$$

Variation of F_b over $\bar{\xi}$ is (18). Variation F_s depending on both ξ and $\bar{\xi}$ is

$$\delta F_s = H_N \int\limits_\Sigma \delta\mathcal{F}_s\, d\,\mathrm{vol}[\Sigma] = H_N \int\limits_\Sigma \frac{\partial \mathcal{F}_s}{\partial \mathcal{D}} \cdot \delta\mathcal{D} d\,\mathrm{vol}[\Sigma] = H_N \int\limits_\Sigma \sigma \cdot \delta\mathcal{D} d\,\mathrm{vol}[\Sigma], \tag{26}$$

where (8) and (13) have been used. Substituting (6) (ommiting second term (see Sec.5)) into (26) and carring out standard integration by parts we get

$$\delta F_s = -H_N \int\limits_\Sigma [\theta(\mathrm{div}\,\sigma, \delta\xi) + \bar{\theta}(\theta(\partial, \sigma(\ ,\partial\bar{\xi})), \delta\bar{\xi})] d\,\mathrm{vol}[\Sigma] + \tag{27}$$

$$H_N \oint\limits_{\partial\Sigma} [\sigma(d\,\mathrm{vol}[\partial\Sigma], \delta\xi) + \sigma(\bar{\theta}(\partial\bar{\xi}, \delta\bar{\xi}), d\,\mathrm{vol}[\partial\Sigma])],$$

where last integral is boundary term.

If external force has potential U, then

$$\delta U = -\int\limits_\Sigma \bar{\theta}(\bar{P}, \delta\bar{\xi}) d\,\mathrm{vol}[\Sigma] - \int\limits_\Sigma \theta(P, \delta\xi) d\,\mathrm{vol}[\Sigma], \tag{28}$$

where $P + \bar{P} = \Pi$ — external force density, P — stretching and $\bar{P}$ — bending parts. Finally, extremality condition $\delta(F + U) = 0$ gives equilibrium equations

$$\Box^2 \bar{\xi}_D - H_N \theta(\partial, \sigma(\ ,\partial\bar{\xi})) = \bar{P} \tag{29}$$

$$H_N \mathrm{div}\,\sigma = -P \tag{30}$$

or in coordinate form:

$$\Box^2 \bar{\xi}^m_D - H_N(\sigma^{\alpha\beta}\bar{\xi}^m_{,\beta})_{,\alpha} = \bar{P}^m; \quad H_N \sigma^{\alpha\beta}_{,\beta} = -P^\alpha,$$

and boundary terms

$$\oint\limits_{\partial\Sigma} \bar{\theta}(H_N \sigma(\partial\bar{\xi}, d\,\mathrm{vol}[\partial\Sigma]) - \theta(\Box\partial\bar{\xi}_D, d\,\mathrm{vol}[\partial\Sigma]), \delta\bar{\xi}) + \tag{31}$$

$$\frac{1}{f+1}\oint\limits_{\partial\Sigma}[f\bar{\theta}(\Box\bar{\xi}_D,\theta(d\,\mathrm{vol}\partial\Sigma),\delta\partial\xi))+\bar{\theta}(\partial^2\bar{\xi}_D(d\,\mathrm{vol}[\partial\Sigma],\delta\partial),\bar{\xi})]+$$

$$H_N\oint\limits_{\partial\Sigma}\sigma(d\,\mathrm{vol}[\partial\Sigma],\delta\xi).$$

The obtained equations (29)-(30) describe selfconsistent system of tangent and orthogonal to $M_{1,3}$ straines. Eq.(29) plays the similar to Einstein equations role: under $\bar{P}=0$ induced by $\bar{\xi}$ geometry is "determined" by the physics, concluded in σ. Eq.(30), in turn, describes 4-dimensional physics, formed by tangent to the plate surface stresses. If one take into account identity of stress tensor σ and energy-momentum tensor T of GR, then (30) can be interpreted as equation of motion of matter in V_4. When $\sigma=0$ we obtain approximative bewave equations (20) of weak bending case.

5 Weakness conditions

Lets clarify meaning of straines weakness condition, which provides validity of Hookes law. One should difer two aspects of the condition.

1. Smallness of strain tensor components lets to use simple quadratic expression (8) for free energy density, which can be interpreted as first member of Taylor decomposition. This condition is expressed by the inequalities with partial derivatives:

$$|\partial\xi|\ll 1;\quad |\partial\bar{\xi}|\ll 1.$$

which means smallness of lengths and angles variation under straining Ξ[10]. Since ξ and $\bar{\xi}$ are independent, then there is two independent smallness parameters: $|\partial\xi|\sim\varepsilon$ and $|\partial\bar{\xi}|\sim\bar{\varepsilon}$. So, algebraic structure of strain tensor (6) is

$$\mathcal{D}\sim\varepsilon+\varepsilon^2+\bar{\varepsilon}^2.$$

Easily to see, that second term is of higher smallness in comparison with the first under any relation between ε and $\bar{\varepsilon}$. Denoting $\varepsilon+\bar{\varepsilon}^2=\delta_1$ we get for the metric (5):

$$g\sim 1+\delta_1.$$

So, in linear approximation all contractions can be made with the help of flat metric θ and $\delta(d\,\mathrm{vol}[\Sigma])$ can be taken zero.

2. Second condition — bending weakness. Fig.1 shows 2-dimensional section of bent plate with magnituded thickness; δl_0 is invariable line element, lying at newtral surface, $\delta l_\pm$ — line elements, lying at opposite sides of the plate and obtained by positive and negative srtetches of δl_0.

[10]Note, that smallness of $\partial\bar{\xi}$ is not so necessary. For example, $\bar{\xi}=\theta(A,x)$ with arbitrary constant vector A determines rigid rotation of $M_{1,3}$ in embedding space.

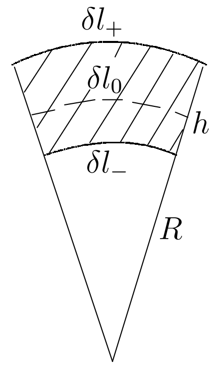

Figure 1: Infinitesemal part of a bent plate.

Hookes law validity means, that relative length variations along thickness h

$$\frac{\delta l_+ - \delta l_-}{\delta l_+ + \delta l_-} \ll 1. \tag{32}$$

It is easily to find, that

$$\frac{\delta l_+}{\delta l_-} = \frac{R + (h/2)}{R - (h/2)},$$

where R — curvature radius of newtral surface in embedding space. After substituting into (32), we get: $h/R \ll 1$. Using embedding theory relation $1/R^2 \sim (\partial^2 \bar{\xi})^2$, we get for multidimensional plate

$$h^2(\partial^2 \bar{\xi})^2 = \delta_2^2 \ll 1. \tag{33}$$

In other words, we have the set $\{\delta_2^m\}$ of smallness parameters — each determines bending smallness in corresponding extradimension.

Now we can compare contributions of F_b and F_s in total expression F. Namely, $\mathcal{D} \sim \delta_1$, $\sigma \sim E\delta_1$, so for integral (25) we get estimation:

$$F_s \sim EH_N V \delta_1^2,$$

where V — 4-dimensional square (volume) of the plate. For integral (17) we get

$$F_b \sim \mu H_N V \sum_{m=1}^{N} h_m^2 (\partial^2 \bar{\xi}^m)^2 \sim EH_N V \sum_{m=1}^{N} (\delta_2^m)^2.$$

So, the two components of total F has independent smallness rates. In case $(\delta_1/\delta_2^m)^2 \gg 1$ for all m F_b can be ommited and we have 4-dimensional physics in flat space-time. In case $(\delta_1/\delta_2^m)^2 \ll 1$ for any m F_s can be ommited and we have curved vacuum space-time. When $(\delta_1/\delta_2^m)^2 \sim 1$ for some m both the energies should be hold and we have curved space-time with fields and matter. Intermediate situations are possible, when different members of $\{\delta_2^m\}$ have different smallness rates.

6 New treatment of Lagrangian formalism

Obtained results lead to curious interpretation of well known objects of standard field theory. Assume, that in (6) $\bar{\xi} = 0$ and we have 4-dimensional physics in Minkowski space-time.

First of all, lets call attention on analogy of the two expressions

$$\delta F = \int \sigma \cdot \delta\mathcal{D}\, d\,\mathrm{vol}, \quad \text{and} \quad \delta S = \frac{1}{2c}\int T \cdot \delta g\, d\,\mathrm{vol}. \tag{34}$$

where $d\,\mathrm{vol}$ denotes suitable form of volume. The first is general thermodynamic relation, connecting stress tensor with infinitisemal variation of an elastic free energy (see (10) and (13)). The second — is well known rule for calculation of symmetric energy-momentum tensor of fields or matter from its lagrangian density $\mathcal{L}$: $S = \int \mathcal{L}\, d\,\mathrm{vol}$.

This analogy can be understood if one take into account the two points: 1) σ and T have the same physical meaning; 2) tensor σ and consequtely T are generated by tangent straining of plate medium. If so, then free energy density of stretches $\mathcal{F}_s(\mathcal{D})$ and matter lagrangian $\mathcal{L}(q, \partial q)$, depending on dynamical field variables $q \equiv \{q^m\}$ and its derivatives should be identified (up to a dimensional constant): $\mathcal{F} = \kappa\mathcal{L}$. Futhermore, variables q acquire the following mechanical sense: they detrmine the way of description of $M_{1,3}$ tangent straining or in mathematical form: $\mathcal{D} = \mathcal{D}(q)$.

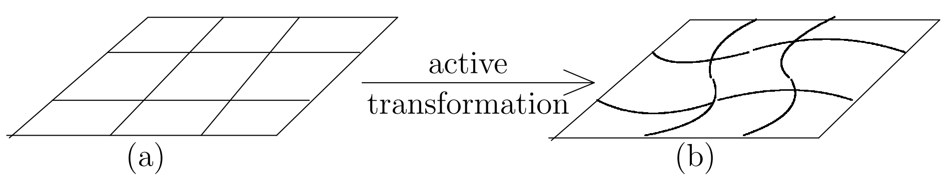

Figure 2: Mechanical interpreting of fields and matter.

When matter is absent, we have unstrained Minkowski plane $M_{1,3}$ (Fig.2(a)) with standard Minkowski metric θ. After diffeomorphism $x \to x' = x'(x)$, which has not passive (as in GR), but active sense of real flat straining of the plate medium we obtain stressed plate $M'_{1,3}$ with curved coordinate lines (Fig.2(b)). In spite of its inner geometry is not change, all metric relations should be represented in general covariant form with using new metric $g = \theta + 2\mathcal{D}$ in accordance with (6). Both $\mathcal{F}$ and $\mathcal{L}$ must implicitly containe metric g (to get from $\mathcal{D}$ or q, and ∂q scalar expressions). Note, that if straining is not strong, then g can be changed by θ in all contractions. From the kind of g we get $\delta g = 2\delta\mathcal{D}$, and so

$$\sigma = \frac{\delta\mathcal{F}}{\delta\mathcal{D}} = 2\frac{\delta\mathcal{F}}{\delta g} \sim \frac{\delta\mathcal{L}}{\delta g} = T. \tag{35}$$

If this analogy is not accident, then *any classical field lagrangian can be treated as free elastic energy density and specific choice of field variables is determined by the kind of plate straining.*

Lets demonstrate it by example with scalar field. Consider the following active transformation $M_{1,3} \to M'_{1,3}$:

$$\theta \to g = (1 + 2\varepsilon_1\phi)\theta, \quad x' = x + \varepsilon_2 a(\phi). \tag{36}$$

where ϕ — dilaton scalar field of small value, a — vector field, depending on coordinates only through ϕ and satisfying the conditions (1) $a(0) = 0$, (2) $\theta(a_\phi, a_\phi) = c = const$ ($a_\phi \equiv da/d\phi$) and (3) $\operatorname{div} a = 0$[11]. First transformation is conformal transformation of metric, which determines homogeneous stretch of $M_{1,3}$, and second — associated with ϕ special (stretchless) translation. Factors ε_1, ε_2 can be taken 0 or 1 independently: they will allow us to distinguish role of dilatation and translation in final expression. Metric g of $M'_{1,3}$ is

$$ds'^2 = g(dx', dx') = (1 + 2\varepsilon_1\phi)\theta(dx + \varepsilon_2 a_\phi d\phi, dx + \varepsilon_2 a_\phi d\phi) = ds^2 + 2\mathcal{D}(dx, dx), \tag{37}$$

where strain tensor

$$\mathcal{D} = \varepsilon_1\phi\theta + \varepsilon_2 a_\phi \dot{\otimes} \partial\phi,$$

and quadratic over ϕ and $\partial\phi$ addends are ommited. Free energy density (8) with using conditions (2) and (3) on a takes the form

$$\mathcal{F} = \frac{\varepsilon_2^2}{2} c\mu(\partial\phi)^2 + 4\varepsilon_1^2(\mu + 2\lambda)\phi^2. \tag{38}$$

Comparing with standard lagrangian of massive newtral scalar field

$$\mathcal{L}_{\text{sf}} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2,$$

we get relation for mass[12] $m^2 \sim \mu + 2\lambda$. When $\varepsilon_1 = 0$, we have only kynetic term. In case $\varepsilon_2 = 0$ we get $\mathcal{F} \sim \phi^2$.

7 Equilibrium equations and conservation laws

It is well known, that extremality of $F[\Xi]$ and $S[q]$ leads to Euler-Lagrange equations, which in turn, provide validity of equilibrium equations in the first case and conservation laws in the second [24, 25]:

$$\delta F = 0 \ \rightarrow \ \operatorname{div}\sigma = 0 \quad \delta S = 0 \ \rightarrow \ \operatorname{div} T = 0; \tag{39}$$

In line of present investigation it would be naturally to use unified language and regard conservation laws as equilibrium equation of some elastic body. In view of results of Sec.4 we can conclude, that *this body is space-time itself.*

[11] Proof of existence of a with conditions (1)-(3) for any ϕ is given in Appendix.
[12] From (9) it follows, that $m = 0$ if $\sigma = 1/N$.

When dynamical system is not closed, we write $\operatorname{div}\sigma = f$, where f — vector of external force density. If f can be represented[13] as $-\operatorname{div}\sigma'$, where σ' — stress tensor of arounding medium, we get

$$\operatorname{div}(\sigma+\sigma') = 0, \tag{40}$$

that is local form of *third Newton law*.

Lets show curious origin of second law in GR in terms of elasticity theory. Assume, that space-time plate is characterized by "phenomenological" multidimensional elastic constant E and σ_P. We'll notate it shortly by $S_g = S_g(E, \sigma_P)$, where S_g — is action for gravitational field, or (up to a dimensional constant) free elastic energy of bending (17). Action of matter we'll notate by S_m. Variation of the full action $S = S_g(E, \sigma) + S_m$ over components of metric g (or 4-dimensional strain tensor $\mathcal{D}$) gives the sum of two tangent (to 4-dimensional space-time) energy-momentum (stress) tensors

$$\delta S = \int (T^{(n)}(E,\sigma) + T^{(t)}) \cdot \delta g d\,\mathrm{vol}. \tag{41}$$

The $T^{(n)}$ is borned by normal straining of the plate, the $T^{(t)}$ — by a tangent ones. Vanishing of (41) gives "generalized" Einstein equations, which in terms of the stress tensors take the form:

$$T^{(n)}(E,\sigma) + T^{(t)} = 0. \tag{42}$$

One can conclude, that Einstein theory operates with *nonstressed* state of space-time. In other words, physical meaning of standard Einstein equations is expressed in intercompensation of tangent stresses, owing to normal and tangent strainings. From the view point of MET this is not necessary equilibrium condition. Equation (42) has been obtained under assuming, that components of g (or $\mathcal{D}$) are dynamical variables. However in elasticity theory such variables are components of strain vector Ξ. This alternative will be discussed below in Sec.(8). True equilibrium equations of the plate are:

$$\operatorname{div}(T^{(n)}(E,\sigma) + T^{(t)}) = 0. \tag{43}$$

Solution of (43) describes equilibrium position of the plate in embedding space and its stress distribution. Lets go in (43) to Einstein GR. As it has been found in [20] and mentioned in Sec.(3) it can be made by setting $\sigma_P = 1/2$. In this condition $T^{(n)}(E, 1/2)$ transforms (up to a constant) to $-G/æ = -({}^4\mathcal{R}\mathrm{ic} - (1/2)g\,{}^4\mathcal{R}_s)/æ$ — pure geometrical Einstein tensor (${}^4\mathcal{R}\mathrm{ic}_{\mu\nu} = {}^4\mathcal{R}^{\lambda}_{\mu\lambda\nu}$ — Ricci tensor), which because of Bianchi identities satisfies $\operatorname{div} G \equiv 0$! Then from (43) automatically follows $\operatorname{div} T^{(t)} \equiv 0$ and we obtain well known statement "equations of a motion are contained in the field equations of GR". Equation $\operatorname{div} T^{(t)} = 0$, when all kind of matter are included in $T^{(t)}$ expresses second Newton law in covariant form. So, we can conclude, that the fundamental principle of classical mechanics can be associated with the *special elastic properties of space-time.*

[13] This is possible, when interaction is local.

8 Problem of variational variables

Previous consideration has led to the question: what relation will be between equations of the theory, obtained from the same functional by variating over different variables? Let us consider it in details.

Let a system of field variables $\{q(x)\}$ considered in region Σ of space-time with metric θ is described by Lagrangian density $\Im(q,\dot{q})$, depending on field variables and its first derivatives over coordinates, shortly abbreviated by $\dot{q}$. Then a variation of action $S = \int\limits_{\Sigma} \Im d\,\mathrm{vol}[\Sigma]$ over variables q will be given by the standard expression:

$$\delta S = \int\limits_{\Sigma} \delta_q \Im \delta q \, d\,\mathrm{vol}[\Sigma] + \int\limits_{\partial\Sigma} \theta(\partial_{\dot{q}} \Im \, \delta q, d\,\mathrm{vol}[\partial\Sigma]) \tag{44}$$

where summation over q and $\dot{q}$ is carried out if necessary, δ_q — Euler-Lagrange operator, which gives field equation $\delta_q \Im = 0$.

Assume, that field variables depend on the set of potentials $\{\xi\}$ through its derivatives ξ' over coordinates: $q = f(\xi')$. Then variations of the field variables take the form: $\delta q = (\partial_{\xi'} f)\delta(\xi')$. Substituting it in (44) we get

$$\delta S = \int\limits_{\Sigma} \delta_q \Im (\partial_{\xi'} f)\delta(\xi') \, d\,\mathrm{vol}[\Sigma] + \int\limits_{\partial\Sigma} \theta(\partial_{\dot{q}} \Im (\partial_{\xi'} f)\delta(\xi'), \, d\,\mathrm{vol}[\partial\Sigma]) = \tag{45}$$

$$- \int\limits_{\Sigma} (\delta_q \Im (\partial_{\xi'} f))' \delta\xi \, d\,\mathrm{vol}[\Sigma] + \int\limits_{\partial\Sigma} \theta((\delta_q \Im)(\partial_{\xi'} f)\delta\xi, d\,\mathrm{vol}[\partial\Sigma]) + \int\limits_{\partial\Sigma} \theta(\partial_{\dot{q}} \Im (\partial_{\xi'} f)\delta(\xi'), d\,\mathrm{vol}[\partial\Sigma])$$

Comparing (44) and (45) we can conclude: any solutions to field equations (44), automatically are solutions to equations (45). The same is valid for boundary conditions and for the case of lagrangians with high derivatives.

Let us demonstrate it by the example with free electromagnetic field. Its lagrangian (up to unessential in vaccum constant) has the kind: $\Im = F \cdot F = F_{\mu\nu}F^{\mu\nu} = 2(\vec{H}^2 - \vec{E}^2)$. Variation of action over $\vec{E}, \vec{H}$ leads to the trivial equation of a motion: $\vec{E} = 0, \vec{H} = 0$. This solutions are simultaneously solutions to more general system of free Maxwell equations, which is obtained by variation of action over 4-dimensional vector-potential.

Returning to GR, we see, that *equations of elasticity theory (29)-(30) together with the boundary conditions (31) has more generality then Einstein Equations from the viewpoint of variational procedure.* So, what is called "exact solutions" is only subset of solution of equilibrium equations, describing nonstressed state of space-time.

9 Matter, its energy-momentum tensor and signature of space-time.

Multidimensional elasticity approach throws new light on meaning of matter, its energy momentum-tensor and energy density.

At first lets clarify geometric nature of an energy-momentum tensor. From the view point of second Newton law for point particle:

$$\vec{a} = \frac{\vec{F}}{m}, \tag{46}$$

force is vector, since acceleration $\vec{a} = \vec{r}_{tt}$ transforms as radii-vector. On the other hand, from the view point of nature of fundamental forces of classical mechanics, force is 1-form F, that follows from its relation with potential U: $F(dx) = dU$. So, equation (46) should be written in the following more exact form:

$$\vec{a} = \frac{g^{-1}(\ ,F)}{m},$$

where g^{-1} — contravariant metric. Geometrical nature of hypersurface element $d\,\mathrm{vol}[\partial\Sigma]$ is clear from its local coordinate representation:

$$d\,\mathrm{vol}[\partial\Sigma]_\mu = \frac{\sqrt{\det[g]}}{(n-1)!}\varepsilon_{\mu\mu_1\ldots\mu_{n-1}}dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{n-1}} \tag{47}$$

where n — dimension of Σ. (47) shows, that $d\,\mathrm{vol}[\partial\Sigma]$ is 1-form too. So, from the definition (11) of stress tensor σ we get that last is linear operator $\hat{\sigma}$, acting in space of 1-forms with tensor structure (1,1)[14].

Lets take for example energy-momentum tensor of isotropic perfect fluid:

$$T = (p+\varepsilon)u\otimes u - pg, \tag{48}$$

where u — 1-form of 4-velocity ($u = g(\vec{u},\ )$), p, ε — scalars of pressure and energy density in rest reference frame. In this frame energy-momentum affinnor $\hat{T}$ has the following kind:

$$\|\hat{T}\| = \mathrm{diag}(\varepsilon, -p, -p, -p). \tag{49}$$

In GR the statement "there is a matter in some region Σ of space-time" is equivalent to the nonequality $\varepsilon > 0,\ x\in\Sigma$. Value p, depend on specific macroscopic properties of matter and can be determined from equation of state.

As it has been discussed above, energy-momentum tensor of a matter in terms of MET characterizes tangent stresses of space-time. So, we go to the natural conclusion, that *energy density of a matter is nothing more no less then a pressure of the plate*

[14] (47) can be treated as dual conjugation of hyphervolume $dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{n-1}}$, so that σ become fully covariant tensor of valency n. We'll use usual (dually conjugated) representation.

medium in time-like direction, while p — is common pressure in space-like ones. From the kind of (49) the following fact can be deduced: *signs of pressures in time-like and space-like directions are opposite.* With the understanding, that pressure is positive, if element of a volume expands under action of the pressure forces (matter tends to fly apart), positive energy density will corresponds to the negative pressure. Its negative signs means, that substance of a plate undergoes contraction along time-like direction. In terms of 4-dimensional world we have bounded state of a matter and anisotropic properties of space-time in time-like and space-like direction. This pressures anisotropy obviously is own to signature $(+1, -1, -1, -1)$ of local pseudoeuclidian metric, or

$$\|\theta\| = \mathrm{diag}(+1, -1, -1, -1) \;\longrightarrow \|\hat{T}\| = \mathrm{diag}(\varepsilon, -p, -p, -p). \tag{50}$$

However, *physically it would be more properly to determine the time-like direction as one, along which the pressure of a plate has a negative sign.* It means that arrow in (50) should be reversed. So, local hyperbolity of space-time, instead of its inducing by metric of embedding space-time, as it has been assumed in Sec.2 (expr. (1)), can be treated as manifestation of signature anisotropy of tangent stresses tensor in different 4-dimensional directions.

Note, that fully isotropic (in 4-dimensional sense) liquid, which is called *false vacuume*, is widely used in cosmology of early Universe [26]. It is described by equation of state $p+\varepsilon = 0$, and energy-momentum tensor $\hat{T}^{\mu}_{\nu} = -p\delta^{\mu}_{\nu}$, which can be intrepreted as common pascalian stress tensor in 4-dimensional euclidian space, without time.

Note also, that such physical definition of time-like direction nothing tells us about arrow of time, as it takes place in special and general relativity too.

10 Boundary conditions

Majority works on field theory use freedom in definition of lagrangian. Namely, two lagrangians, differing by total divergence it is practice to consider equivalent [24, 25]. Particulary, in variational procedure of GR, total divergence terms, including second derivatives of metric, are usually rejected [24]. This is motivated by vanishing of a metric variations at a bound of variation region. As a matter of a fact, this requirement is perfectly unnecessary from the mathematical viewpoint and has purely physical nature. It is obviously, that adding of divergence $\mathrm{div}\mathcal{V}$ to a free energy density $\mathcal{F}$ will change boundary conditions:

$$\mathcal{F} \to \mathcal{F}' = \mathcal{F} + \mathrm{div}\mathcal{V} \text{ means that } F' = F + F_{\mathrm{surf}} = \int\limits_{\Sigma} \mathcal{F}\, d\,\mathrm{vol}[\Sigma] + \int\limits_{\partial\Sigma} \theta(\mathcal{V}, d\,\mathrm{vol}[\partial\Sigma]). \tag{51}$$

From the kind of last integral one can note, that vector $\mathcal{V}$ has meaning of *vector surface energy density*, which physically can be related with boundary effects (such as, for

example, surface tension of liquid drop). Stokes theorem lets write down this energy in the same footing as volume energy. Requirement of vanishing of variations at the bound means, that certain choise of boundary condition is fixed. In case of plate straining, conditions $\delta\Xi|_{\partial\Sigma} = 0$ and $\delta(\partial\Xi)|_{\partial\Sigma} = 0$ mean, that edges of the plate are *pinned.* This choise is typical for modern field theory. Clearly, that this is not unique choise. Vanishing of the first variation of free energy, gives, besides of equilibrium equations, boundary conditions (31), which can be satisfied by different ways. This conditions will determine the values of integration constants and a spectrum of eigen values in Sturm-Liouville task. The most simple boundary conditions at the plate edges become in cases of *pinned, simply supported or free bounds* [23]. The case of unbounded in one or several dimensions plate is also sufficiently simple.

However, such arbitrary choice of boundary conditions is speculative, or at best, model. *The problem of boundary conditions must be disolved by an experimental way, if we are talking about physical theory.* So, the role of dynamical equations should be expanded: its solutions after comparing with observational dates can be used not only for describing of dynamics of gravitational and other fields, but also for investigation of boundary conditions of our world. In paper [22] the case of rectangle plate with different combinations of boundary conditions has been considered in cosmological aspects. It has been shown, that measurable 4-dimensional geometric invariants are sensible (in some extent) to a choise of boundary conditions.

It is remarkable, that gauge fixing in gauge theories concerns to choise of boundary conditions too. It can be easily seen in the simplest case of $U(1)$-symmetry of electrodynamics. Action of interaction $S_{\rm int}$ in electrodynamics has the form

$$S_{\rm int} = \int\limits_{\Sigma} A(j)\, d\,{\rm vol}[\Sigma], \tag{52}$$

where A and j — 1-form of potential and vector of electric current correspondingly. Let make gauge transformation $A \to A + \nabla\chi$, where χ — arbitrary smooth scalar function. Then (52) transforms to

$$S'_{\rm int} = S_{int} + \int\limits_{\Sigma} d\chi(j)\, d\,{\rm vol}[\Sigma] = S_{\rm int} + \int\limits_{\Sigma} {\rm div}\,(\chi j)\, d\,{\rm vol}[\Sigma] - \int\limits_{\Sigma} \chi {\rm div}\, j\, d\,{\rm vol}[\Sigma].$$

Vanishing of the last integral gives conservation law of charge: ${\rm div} j = 0$, while second integral can be transformed into surface term:

$$\oint_{\partial\Sigma} \chi\theta(j, d\,{\rm vol}(\partial[\Sigma])).$$

Its vanishing means special choise of boundary conditions for vector χj, playing role of surface energy density. So, gauge invariance of action (and conservation of electric charge) is closely related with boundary conditions for fields and its sources.

11 Conclusion

Proposed approach gives unified base for description of gravity, field and matter. As standard elasticity theory, the approach has apparently "phenomenological" kind, since 1) postulate about Hookes law validity is accepted; 2) phenomenological Lame coefficients should be measured experimentally. Quotation-marks mean, that this multidimensional phenomenologicity is manifested as fundamental laws of 4-dimensional world (see Eq.(29), (30)). From the other hand, any classical field theory (for example Einstein GR) can be treated as phenomenological, since 1) it postulates lagrangian (in view (MET) — free elastic energy functional), whose simple kind has the similar to Hookes law nature; 2) it postulates set of fundamental constants, plaing role of Lame coefficient. So, theories of gravity with nonlinear over curvature terms, can be regarded as MET with generalized Hookes law.

There is many deep analogies of MET with modern fundamental theories, mentioned in Introduction. Detail discussion we defer to the future.

Lets shortly discuss problems of motion and quantization. Motion of any finite body in 4-dimensional world can be represented by its world tube. From the view point of MET this last should be considered as *thin elastic bar*, generally speaking, *bent* and *twisted.* Bending is determined by acceleration of mass center, twisting — by rotation of the body. We can accept the following natural hypothesis: *standard mechanical action* $S = \int \mathcal{L}\, dt = \int (K - U)\, dt$, *where* K *and* U *— kynetic and potential energy correspondingly has the sense of free energy of bending and twisting of the elastic bar.* Preliminary investigation supports this idea. Another possibility, which indirectly follows from the previous consideration, is related to the theory of *nemathic medium.* Anisotropy of energy-momentum tensor in time-like and space-like directions can be regarded as manifestation of *nemathic properties of space-time plate.* This point of view demands multidimensional generalization of standard nemathic theory (see [6]).

Problems of time arrow and of observer's motion along time-like world line remain opened (as in GR and majority of its generalization). Presumably, it can'nt be resolved in classical physics (see [19]).

It would be naturally to believe, that quantum phenomena should be related to a small finite size of the plate thicknesses. One can consider the set of thicknesses $\{h_m\}$ as a set of scales, which *determine elementary stable stressed states of different sort, observing as elementary particles.* This idea is closely related with results of work [5].

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A Existing of vector field a

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Lets prove existence of vector field a, satisfying the following conditions:

1) a depends on coordinate only through scalar field ϕ: $a = a(\phi)$;

2) $a(0) = 0$;

3) $\theta(a_\phi, a_\phi) = const$, where $a_\phi \equiv da/d\phi$, θ — Minkowski metric.

4) $\operatorname{div} a = 0$.

Since all conditions (1)-(4) are coordinate independent, we can work in any coordinate system. Lets begin from point (4). After changing coordinates[15]: $x'^0 = \phi(x)$, $x'^{1,2,3} = x^{1,2,3}$, condition (4) will take the form (we ommit "$'$"):

$$\operatorname{div} a = d\phi(a_\phi) = a^0_\phi = 0.$$

So, (4) will be satisfied, when $a = (c_0, \vec{a})$ in the special coordinate system ($\vec{a} = (a^1, a^2, a^3)$, a^i — any differentiable functions of ϕ, $c_0 = const$). Put $c_0 = 0$, then (3) means $\vec{a}^2_\phi = -(a^1)^2 - (a^2)^2 - (a^3)^2 = -R^2 = const$. Going to spherical angles $\vartheta = \vartheta(\phi)$, $\varphi = \varphi(\phi)$, we get:

$$\begin{cases} a^1_\phi = R\sin\theta\cos\varphi; \\ a^2_\phi = R\sin\theta\sin\varphi; \\ a^3_\phi = R\cos\theta, \end{cases} \text{and integrating} \begin{cases} a^1 = R\int\sin\theta\cos\varphi\, d\phi + c_1 = RF_1(\phi) + c_1; \\ a^2 = R\int\sin\theta\sin\varphi\, d\phi + c_2 = RF_2(\phi) + c_2; \\ a^3 = R\int\cos\theta\, d\phi + c_3 = RF_3(\phi) + c_3. \end{cases}$$

Putting $c_i = -RF_i(0)$ we satisfy remained condition (2).

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